Woodseaves C.E Academy

Progression in Written Calculations

To be read in conjunction with the 'Progression in Mental Calculation Skills Policy'

Contents

Introduction

Aims of the Written Calculation Policy

This calculation policy has been produced to ensure consistency and progression in teaching throughout the school. It aims to give an overview of the key written calculation strategies that will be taught. The policy aims to identify the progression in each of the four operations that children will typically follow. Each stage builds upon previous experience. It is not intended as a straightjacket, nor is it a scheme of work. It recognises that children will develop their mathematical skills at different rates and have their own individual learning styles. They will develop calculation skills through a combination of practical, oral and mental activities. Although the focus of this policy is on pencil and paper procedures, it is important to recognise that in every written method there is an element of mental processing. Written calculation strategies will therefore be taught alongside mental calculation strategies and should be seen as complementary to and not as separate from them. Informal written recording will take place regularly and is an important part of learning and understanding. More formal written methods follow only when the child is able to use a wide range of mental calculation strategies.

The Importance of Decision Making

The emphasis of our teaching will always be to facilitate understanding and not simply to arrive at a correct answer. Children will always be encouraged to look at a calculation/problem and then decide which is the best method to choose. Our aim is for children to be able to select an efficient method of their choice (whether this be mental or written) that is appropriate for a given task. They will do this by always asking themselves: • 'Can I do this in my head?' • 'Can I do this in my head using drawings or jottings?' • 'Do I need to use a pencil and paper procedure?' • 'Do I need a calculator?'

Finally….

The overall aim is that when children leave Woodseaves C.E Academy, they:

• have a secure knowledge of number facts and a good understanding of the four operations

• are able to use this knowledge and understanding to carry out calculations mentally and to apply appropriate strategies when using larger numbers

• have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;

N.B This policy takes account of the fact that children may move through this progression at different rates but it does make clear where children need to be by the end of Year 2 and Year 6 if they are to meet or exceed age-related expectations.

Maths Vocabulary

Children should experience the various maths vocabulary used within the four operations for mental and written questions.

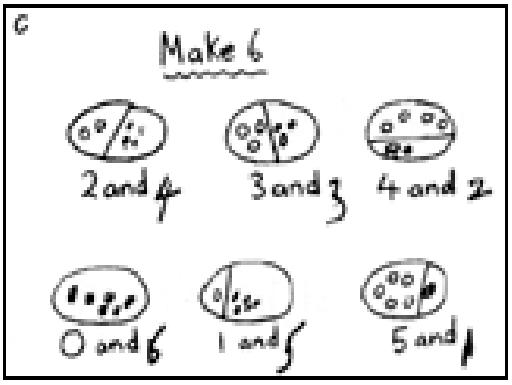
Symbol	Words Used
+	Addition, add, sum, plus, increase, total, more, together, and
-	Subtraction, subtract, minus, less, difference, decrease, take away, deduct, fewer, take from, reduce
×	Multiplication, multiply, product, multiplied by, times, lots of, groups of, times tables
÷	Division, divide, divided by, share, share equally, divisible by, divide into, group, quotient (is the result of division)

PROGRESSION THROUGH CALCULATIONS FOR ADDITION

MENTAL CALCULATIONS (See Progression in Mental Skills Booklet)

Stage 1

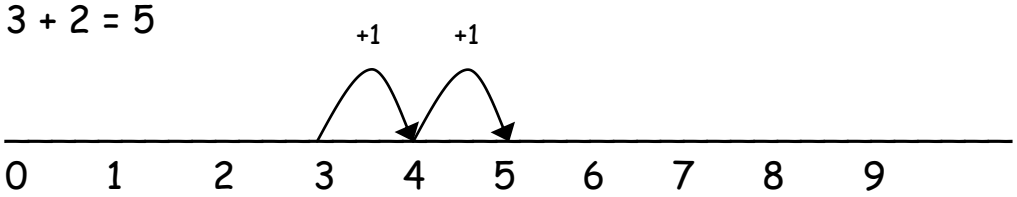
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, rhymes, songs etc. Pictoral representations are used.



Stage 2

They use number lines and practical resources to support calculation and teachers *demonstrate* the use of the number line.

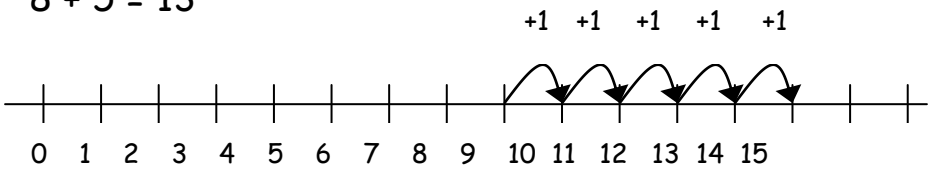
3 + 2 = 5



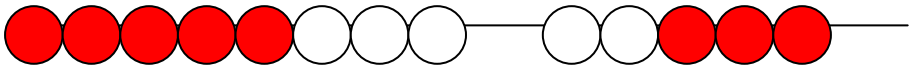
This stage could also be supported with a number grid/100 square

Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

8 + 5 = 13



Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

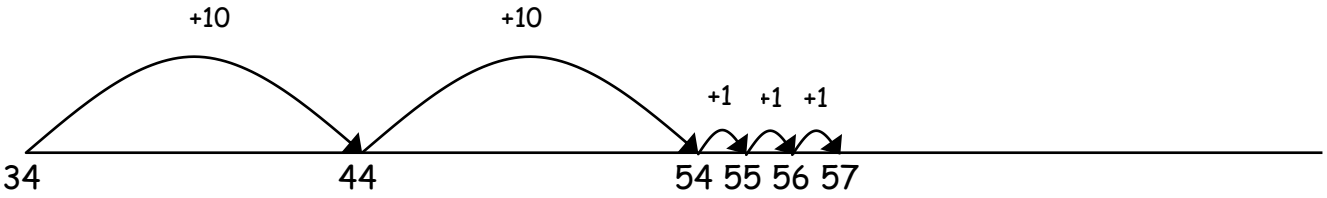


Stage 3

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

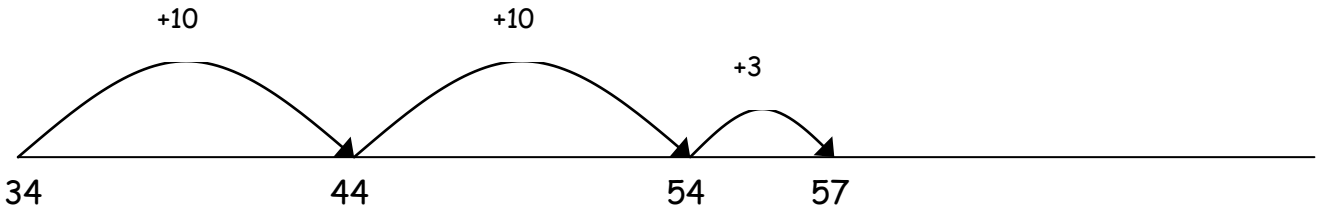
✓ First counting on in tens and ones.

34 + 23 = 57



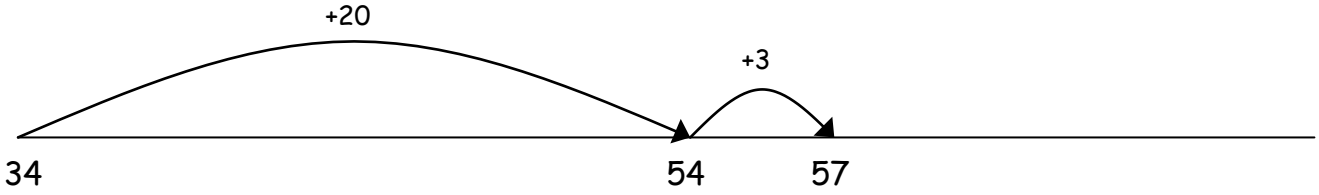
✓ Then helping children to become more efficient by adding the units in one jump (by using the known fact 4 + 3 = 7).

34 + 23 = 57



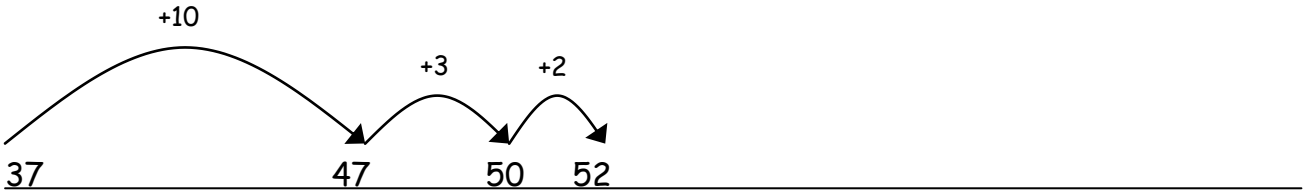
✓ Followed by adding the tens in one jump and the units in one jump.

34 + 23 = 57



✓ Bridging through ten can help children become more efficient.

37 + 15 = 52

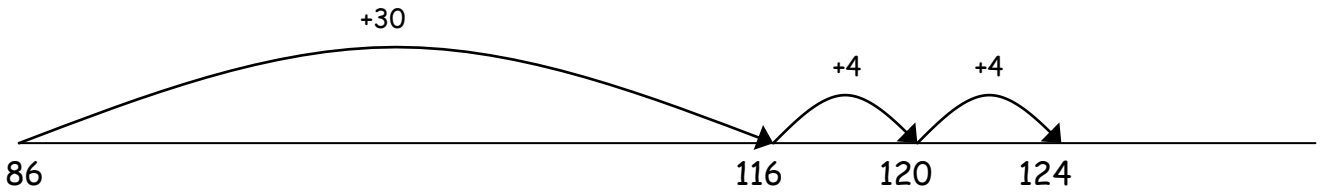


Stage 4 (By the end of Year 2)

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

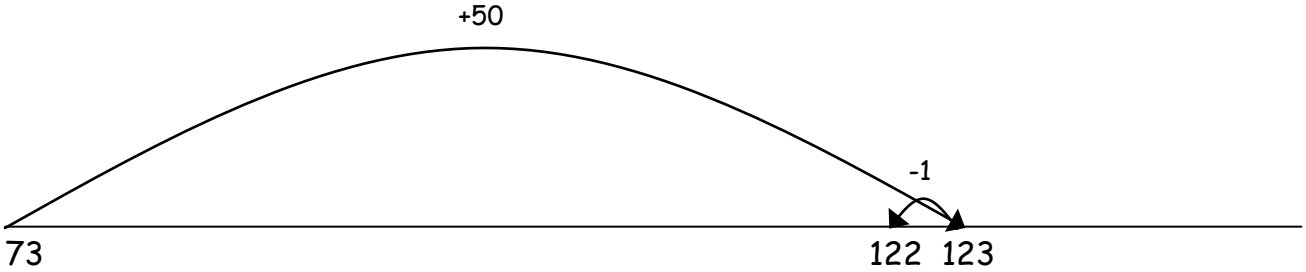
- ✓ Count on from the largest number irrespective of the order of the calculation.

38 + 86 = 124



- ✓ Compensation by rounding to the nearest 10 and take away the number they had added when rounding.

49 + 73 = 122



(Year 3 onwards)

Stage 5 (Pre- standard written method)

Formal Expanded Methods - Addition uses partitioning.

Adding most significant digits first then, moving to adding least significant digits.

```
  67
+ 24
  80  (60 + 20)
  11  ( 7 + 4)
  91
```

```
  267
+  85
  200
   140  (60 + 80)
    12  ( 7 + 5)
  352
```

Moving to adding the least significant digits first in preparation for 'carrying'.

```
  67
+ 24
  11  ( 7 + 4)
  80  (60 + 20)
  91
```

```
  267
+  85
   12  ( 7 + 5)
  140  (60 + 80)
  200
  352
```

Adding the least significant digits first

```
  67
+ 24
  11  ( 7 + 4)
  80  (60 + 20)
  91
```

```
  267
+  85
   12  ( 7 + 5)
  140  (60 + 80)
  200
  352
```

Stage 6 Introducing the standard written method

Addition using the standard column method (short method)

From this, children will begin to carry below the line.

625	783	367
+ 48	+ 42	+ 85
673	825	452
1	1	1 1

Using similar methods, children will:

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;*
- ✓ *know that the decimal points should line up under each other, particularly when adding mixed amounts, e.g. £3.59 + 78p.*

Stage 7

Children should extend the carrying method to numbers with at least four digits.

587	3587
+ 475	+ 675
1062	4262
1 1	1 1 1

Using similar methods, children will:

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to three digits and the same number of decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m – 280 cm.*

Stage 8

Children should extend the carrying method to calculations with any number of digits.

7648	6584	42
+ 1486	+ 5848	6432
9134	12432	786
1 1 1	1 1 1	3
		+ 4681
		11944
		1 2 1

(By Year 6)

Using similar methods, children will

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to four digits and either one or two decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding mixed amounts, e.g. 401.2 + 26.85 + 0.71.*

When using mixed numbers children need to understand that place holders do not change the value of the number.

401.20 ←

26.85

0.71

428.76

1

The children need to be aware that it does not change the value of the number, just supports with lining up the place value.

+ - + - + - + - + - + - +

By the end of all the stages, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- they are not ready i.e. lack of understanding of place value.
- they are not confident with previous stages.

PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

Stage 1

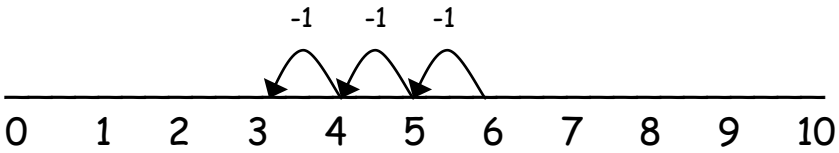
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.

Stage 2

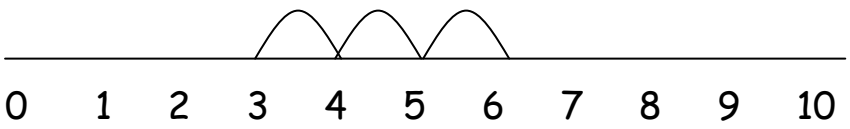
They use number lines and practical resources to support calculation. Teachers *demonstrate* the use of the number line.

This stage could also be supported with a number grid/100 square

6 – 3 = 3

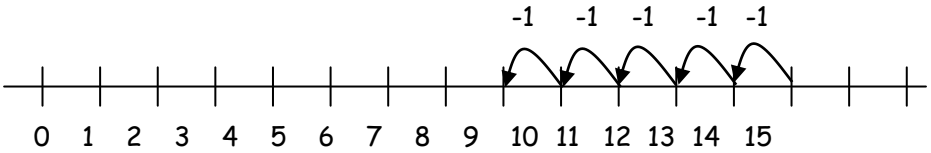


The number line should also be used to show that 6 - 3 means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.



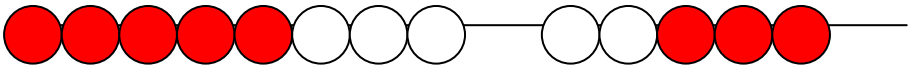
Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

13 – 5 = 8



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

13 – 5 = 8



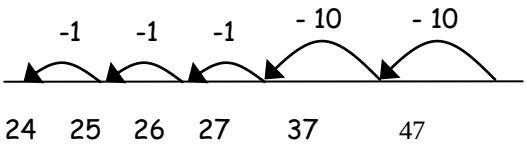
Stage 3

Children will begin to use empty number lines to support calculations with 2 digit numbers

Counting back

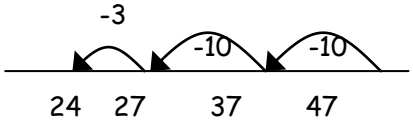
✓ First counting back in tens and ones.

47 – 23 = 24



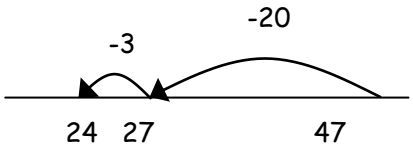
✓ Then helping children to become more efficient by subtracting the units in one jump (by using the known fact 7 – 3 = 4).

47 – 23 = 24



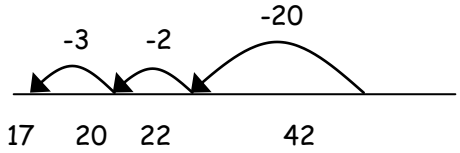
✓ Subtracting the tens in one jump and the units in one jump.

47 – 23 = 24



✓ Bridging through ten can help children become more efficient.

42 – 25 = 17



<u>Stage 4</u> (By the end of Year 2)

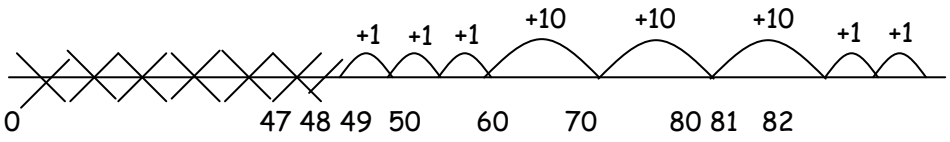
Counting on – difference between the numbers

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.

82 - 47



Help children to become more efficient with counting on by:

✓ Subtracting the units in one jump;
✓ Subtracting the tens in one jump and the units in one jump;
✓ Bridging through ten.

Children will continue to use empty number lines with increasingly large numbers.

Year 3 onwards
Stage 5 Pre- standard written method

Partitioning and decomposition

This process can be demonstrated using arrow cards to show the partitioning and base 10 materials to show the decomposition of the number.

NOTE When solving the calculation 89 – 57, children should know that 57 **does NOT EXIST AS AN AMOUNT** it is what you are subtracting from the other number. Therefore, when using resources, children would need to count out only the 89.

$$\begin{array}{rcl} 89 & = & 80 + 9 \\ -\ 57 & & \underline{50 + 7} \\ & & 30 + 2 = 32 \end{array}$$

Initially, the children will be taught using examples that do not need the children to exchange.

From this the children will begin to 'take from the next column'.

$$\begin{array}{r} 71 \\ -\ 46 \\ \hline \end{array}$$

Step 1
$$\begin{array}{r} 70 + 1 \\ -\ \underline{40 + 6} \end{array}$$

Step 2
$$\begin{array}{r} 60 + 11 \\ -\ \underline{40 + 6} \\ 20 + 5 = 25 \end{array}$$

The calculation should be read as e.g. take 6 from 1.

This would be recorded by the children as

$$\begin{array}{r} \overset{60}{\cancel{70}} + {}^{1}1 \\ -\ \underline{40 + 6} \\ 20 + 5 = 25 \end{array}$$

Children should know that units line up under units, tens under tens, and so on.

Stage 6

Partitioning and decomposition H T O's

754 =
- 86

Step 1 700 + 50 + 4
- 80 + 6

Step 2 700 + 40 + 14 *(adjust from T to O)*
- 80 + 6

Step 3 600 + 140 + 14 *(adjust from H to T)*
- 80 + 6
600 + 60 + 8 = 668

This would be recorded by the children as

600 140
~~7~~00 + ~~5~~0 + ^{1}4
- 80 + 6
600 + 60 + 8 = 668

Stage 7

Decomposition – standard method

614 1
~~7~~54
- 286
468

5 13 1
~~6~~~~4~~67
- 2684
3783

Children should:

- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *know that decimal points should line up under each other.*
- ✓ *be able to subtract with 'taking from the next column'*

Stage 8

Partitioning and Decomposition – decimal numbers

- ✓ *using this method, children should also begin to find the difference between decimal numbers*
- ✓ *know that decimal points should line up under each other.*

For example:

89.5 = 80 + 9 + 0.5
-43.8 - 40 + 3 + 0.8

= 80 + 8 + $0.^{1}5$ *(adjust from T to O)*
80.85
- 40 + 3 + 0.8
40.38
40 + 5 + 0.7

= 40.57

Children should:

- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places;*
- ✓ *know that decimal points should line up under each other.*

Stage 9 (By end of Year 6)

NB If children have reached the concise stage they will then continue with this method. They will not go back to using the expanded methods.

Decomposition – standard method (decimals)

5 13 1
~~6~~ ~~4~~ 6.7
- 268.4
378.3

Children should:

- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;*
- ✓ *know that decimal points should line up under each other.*

- ✓ *Using this method, children should also begin to find the difference between two three-digit sums of money, with or without taking from the pounds to the pence.*

For example:

£8.^{8}9^{1}5
-£4. 3 8
£4. 57

+ - + - + - + - + - + - +

By the end of all the stages, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

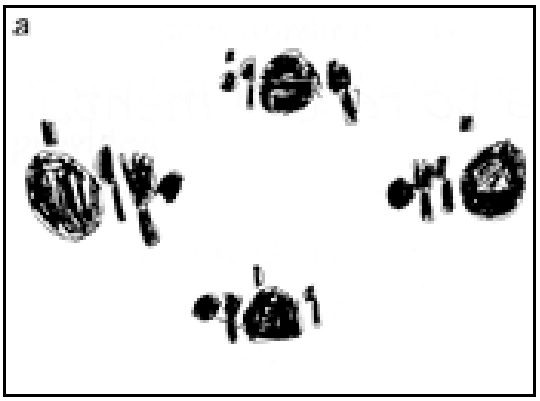
Children should not be made to go onto the next stage if:

- they are not ready i.e. lack of understanding of place value.
- they are not confident with previous stages

PROGRESSION THROUGH WRITTEN CALCULATIONS FOR MULTIPLICATION

Stage 1

Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.



Children need visual examples to understand multiplication. Children can group items in a picture or solid items in dishes. When confident with grouping items together, they can then move on to Stage 2 by using numbers as well as pictures to link the two together the two stages together

Stage 2

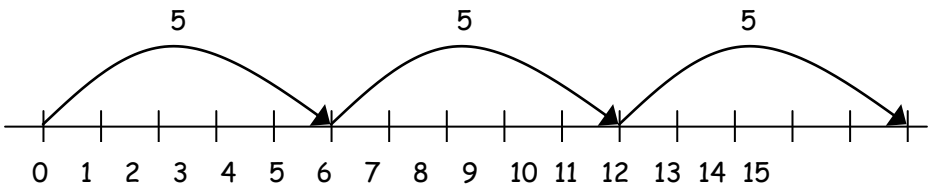
Children will develop their understanding of multiplication and use jottings to support calculation:

✓ **Repeated addition**

3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3

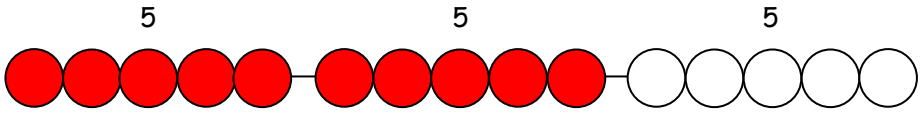
Repeated addition can be shown easily on a number line:

5 x 3 = 5 + 5 + 5



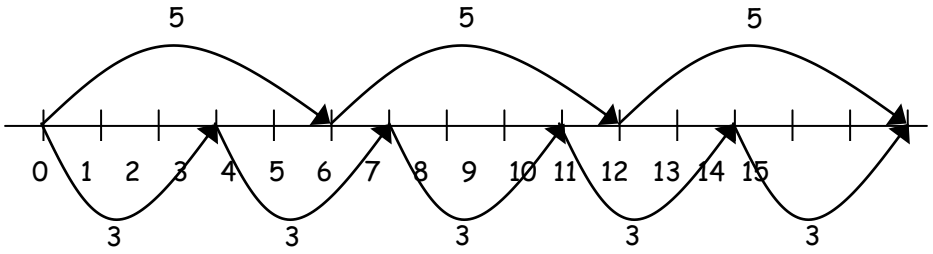
and on a bead bar:

5 x 3 = 5 + 5 + 5



Stage 3

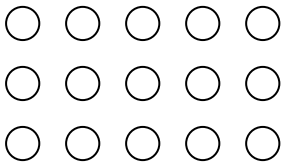
Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on the number line.



Stage 4

✓ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



5 x 3 = 15

3 x 5 = 15

Stage 5

Children will also develop an understanding of

- ✓ **Scaling**

e.g. Find a ribbon that is 4 times as long as the blue ribbon

Stage 6 (By End of Year 2)

- ✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

□ x 5 = 20 3 x △ = 18 □ x 4 = 32

Stage 7

- ✓ **Partitioning**

38 x 5 = (30 x 5) + (8 x 5)
= 150 + 40
= 190

Stage 8

Grid method

TO x O
23 x 8

Children will approximate first
23 x 8 is approximately 25 x 8 = 200

x	20	3
8	160	24

160
+ 24

184

Grid method

HTO x O

346 x 9

Children will approximate first
346 x 9 is approximately 350 x 10 = 3500

x	300	40	6
9	2700	360	54

2700
+ 360
+ 54
3114
1 1

TO x TO

72 x 38
Children will approximate first
72 x 38 is approximately 70 x 40 = 2800

x	70	2
30	2100	60
8	560	16

2100
+ 560
+ 60
+ 16
2736
1

Grid Method

HTO x TO

372 x 24

Children will approximate first
372 x 24 is approximately 400 x 25 = 10000

x	300	70	2
20	6000	1400	40
4	1200	280	8

6000
+ 1400
+ 1200
+ 280
+ 40
+ 8
8928
1

Stage 9

Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other.

e.g. 4.9 x 3

Children will approximate first
4.9 x 3 is approximately 5 x 3 = 15

x	4	0.9
3	12	2.7

12
+ 2.7
14.7

Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.

For example:

4.92 x 3

Children will approximate first
4.92 x 3 is approximately 5 x 3 = 15

x	4	0.9	0.02
3	12	2.7	0.06

$$\begin{array}{r} 12 \\ +\ 0.7 \\ +\ 0.06 \\ \hline 12.76 \end{array}$$

Stage 10

Short Multiplication

The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.

$$\begin{array}{r} 38 \\ \times\ 7 \\ \hline 266 \\ 5 \end{array}$$

Children need to understand that the number they are multiplying by, multiplies into each number starting with the lowest place value.

Stage 11 (By Year 6)

Long Multiplication

56 × 27 is approximately 60 × 30 = 1800.

```
   56
X  27
 1120    56 x 20
  392    56 x  7
 1512
  1
```

- Children need to remember that the number they are multiplying by, multiplies into each number starting with the lowest place value.
- Children start with the tens then move onto the units.
- Children can place a zero in the units place of the tens row but they must be aware that the row is multiply of 10

Children who are already secure with multiplication for TO × O and TO × TO should have little difficulty in using the same method for HTO × TO.

286 × 29 is approximately 300 × 30 = 9000.

```
   286
X  29
 5720    286 x 20
 2574    286 x  9
 8294
 1
```

This method can be used for decimals by making the numbers being used into whole numbers first by multiplying by 10 and then dividing the answer the answer by 10.

i.e

8.6 x 6 = 8.6 x 10 = 86 x 6 = 516 ÷ 10 = 51.6

```
   86
X  6
 516    86 x  6
  3
```

+ - + - + - + - + - + - +

By the end of all the stages, children will have a range of calculation methods, mental and written.

Children should not be made to go onto the next stage if:

- they are not ready i.e. lack of understanding of place value.
- they are not confident with previous stages

PROGRESSION THROUGH CALCULATIONS FOR DIVISION

Stage 1

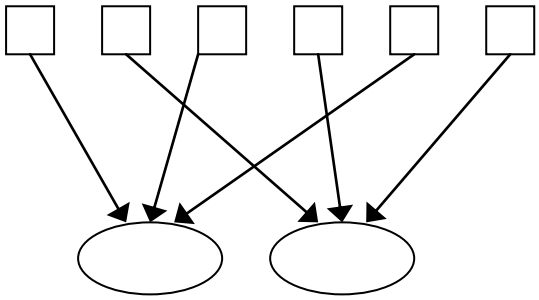
Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

Stage 2

Children will develop their understanding of division and use jottings to support calculation

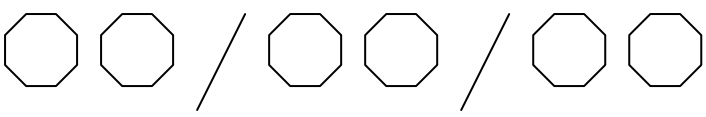
✓ **Sharing equally**

6 sweets shared between 2 people, how many do they each get?



✓ **Grouping or repeated subtraction**

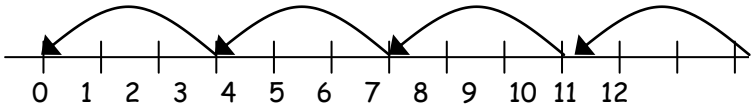
There are 6 sweets, how many people can have 2 sweets each

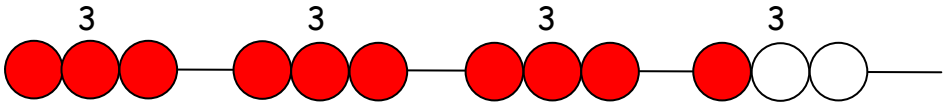


Stage 3

✓ **Repeated subtraction using a number line or bead bar**

12 ÷ 3 = 4

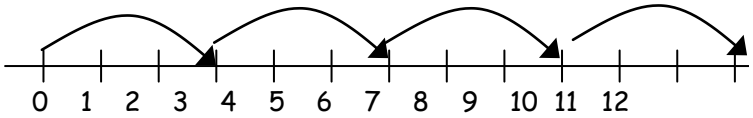


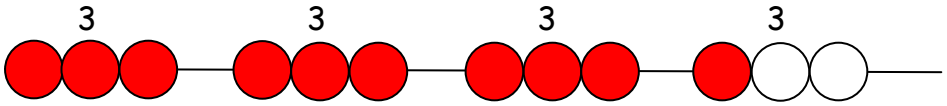


Or

✓ **Repeated addition using a number line or bead bar**

12 ÷ 3 = 4





The bead bar will help children with interpreting division calculations such as 10 ÷ 5 as 'how many 5s make 10?'

Stage 4 (By end of Year 2)

Missing numbers – using the inverse

✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

□ ÷ 2 = 4 20 ÷ △ = 4 100 ÷ △ = 4

Stage 5

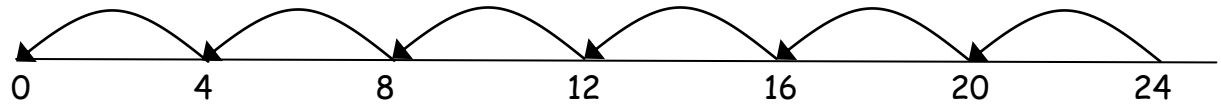
Ensure that the emphasis is on grouping rather than sharing.

Children will continue to use:

- ✓ **Repeated subtraction or addition using a number line**

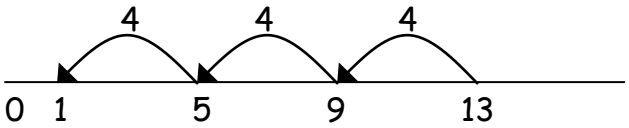
Children will use an empty number line to support their calculation.

24 ÷ 4 = 6



Children move onto calculations involving remainders.

13 ÷ 4 = 3 r 1



Stage 6

Chunking Method

256 ÷ 7 =

	140	(20 x 7)
	70	(10 x 7)
Running Total	210	
	35	(5 x 7)
Running Total	245	
	7	(1 x 7)
Final Total	252	36

Answer 36 remainder 4
Check (36 x 7) +4 = 256

Answer: 36 remainder 4 or $36\ ^{4}/_{7}$

Chunking Table

X 7

20 x 7 = 140
10 x 7 = 70
5 x 7 = 35
2 x 7 = 14
1 x 7 = 7

By doing a chunking table the children have already got the chunks ready to use.

Stage 7

Remind children that this is the only time they will start with the highest place value.

Short division

284÷ 6

$$6\overline{)284} = 47\ r2$$

Answer 47 r 2

Remainder as a fraction

47 r2

$47\frac{2}{6}$ or $47\frac{1}{3}$

Remainders as Decimals (the question will normally say to how may decimal places)

To one decimal place (dp)

47.3

Stage 8 (By end of Year 6)

Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other.

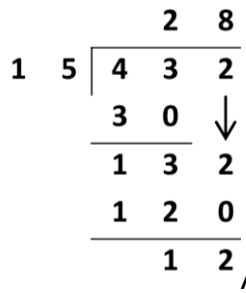
87.5 ÷ 7

$$7\overline{)87.5} = 12.5$$

Answer : 12.5

Stage 9

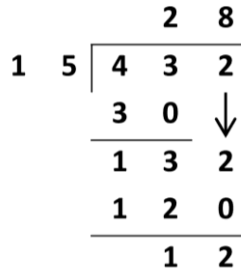
Long Division with remainders



Answer: 28 r ^{12}

Start with the highest place value use the first 2 digits, 43 divided by 15 is 2. 2 goes in the answer, 2 x 15 is 30 that is placed under the first two digits. Then the difference between those numbers is worked out and the next number is brought down to make a new number. Then the process starts again until all numbers have been used. If there is no more numbers to be brought down this is written as a remainder.

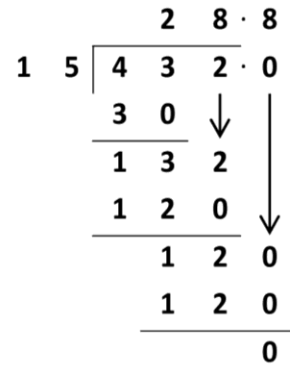
Long Division remainders as a fraction



Using the same process:
The reminder can be changed into a fraction. Simplify the fraction if possible.

$$\frac{\cancel{12}}{\cancel{15}} \quad \frac{4}{5}$$

Long Division remainders as a decimal



Using the same process, instead of leaving the remainder add a decimal point and a zero. The zero then gets brought down to carry on the process to calculate the remainder as a decimal.

Answer: 28·8

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By the end of all stages children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- they are not ready i.e. lack of understanding of place value.
- they are not confident with previous stages